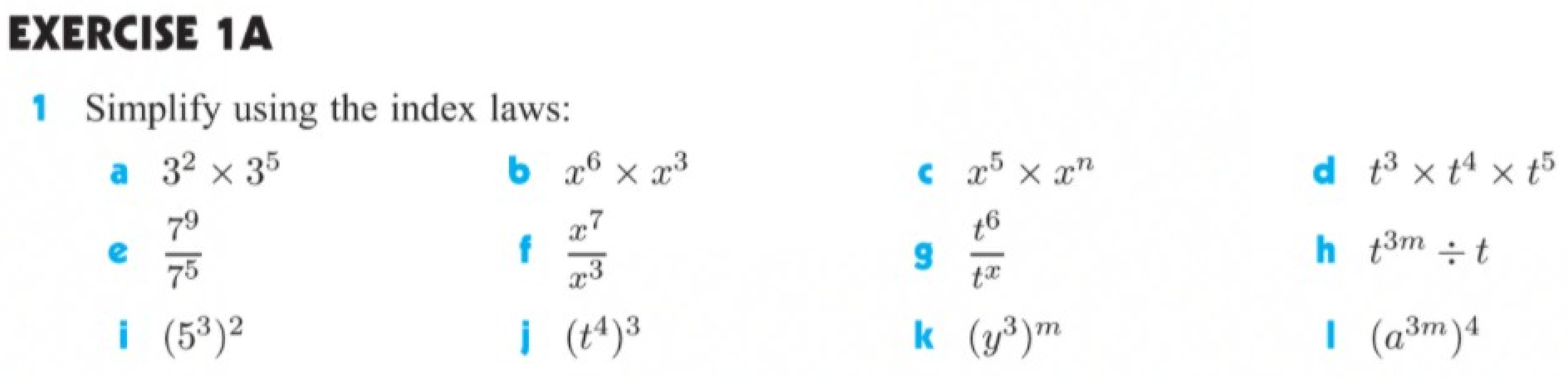
A INDEX LAWS

If the bases a and b are both positive, and the indices m and n are integers, then:

$a^m \times a^n = a^{m+n}$	To **multiply** numbers with the **same base**, keep the base and **add** the indices.
$\dfrac{a^m}{a^n} = a^{m-n}$	To **divide** numbers with the same base, keep the base and **subtract** the indices.
$(a^m)^n = a^{mn}$	When **raising** a **power** to a **power**, keep the base and **multiply** the indices.
$(ab)^n = a^n b^n$	The power of a product is the product of the powers.
$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$	The power of a quotient is the quotient of the powers.
$a^0 = 1, \quad a \neq 0$	Any non-zero number raised to the power of zero is 1.
$a^{-n} = \dfrac{1}{a^n}$	and in particular, $a^{-1} = \dfrac{1}{a}$.



EXERCISE 1A

1 Simplify using the index laws:

a $3^2 \times 3^5$
b $x^6 \times x^3$
c $x^5 \times x^n$
d $t^3 \times t^4 \times t^5$
e $\frac{7^9}{7^5}$
f $\frac{x^7}{x^3}$
g $\frac{t^6}{t^x}$
h $t^{3m} \div t$
i $(5^3)^2$
j $(t^4)^3$
k $(y^3)^m$
l $(a^{3m})^4$

Example 1

Self Tutor

Express in simplest form with a prime number base:

a 9^4

b 4×2^p

c $\frac{3^x}{9^y}$

d 25^{x-1}

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Express in simplest form with a prime number base:

a 9^4 **b** 4×2^p **c** $\dfrac{3^x}{9^y}$ **d** 25^{x-1}

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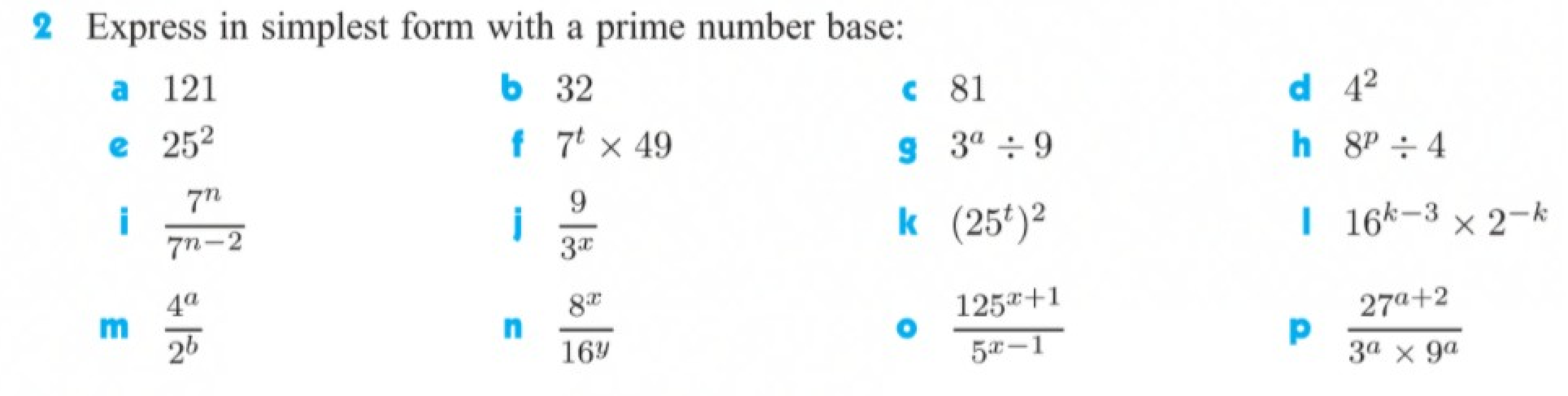
a
$$\begin{aligned} &9^4 \\ &= (3^2)^4 \\ &= 3^{2\times 4} \\ &= 3^8 \end{aligned}$$

b
$$\begin{aligned} &4 \times 2^p \\ &= 2^2 \times 2^p \\ &= 2^{2+p} \end{aligned}$$

c
$$\begin{aligned} &\frac{3^x}{9^y} \\ &= \frac{3^x}{(3^2)^y} \\ &= \frac{3^x}{3^{2y}} \\ &= 3^{x-2y} \end{aligned}$$

d
$$\begin{aligned} &25^{x-1} \\ &= (5^2)^{x-1} \\ &= 5^{2(x-1)} \\ &= 5^{2x-2} \end{aligned}$$



2 Express in simplest form with a prime number base:

- **a** 121
- **b** 32
- **c** 81
- **d** 4^2
- **e** 25^2
- **f** $7^t \times 49$
- **g** $3^a \div 9$
- **h** $8^p \div 4$
- **i** $\dfrac{7^n}{7^{n-2}}$
- **j** $\dfrac{9}{3^x}$
- **k** $(25^t)^2$
- **l** $16^{k-3} \times 2^{-k}$
- **m** $\dfrac{4^a}{2^b}$
- **n** $\dfrac{8^x}{16^y}$
- **o** $\dfrac{125^{x+1}}{5^{x-1}}$
- **p** $\dfrac{27^{a+2}}{3^a \times 9^a}$

Example 2

Self Tutor

Remove the brackets of:

a $(2x)^3$

b $\left(\dfrac{3c}{b}\right)^4$

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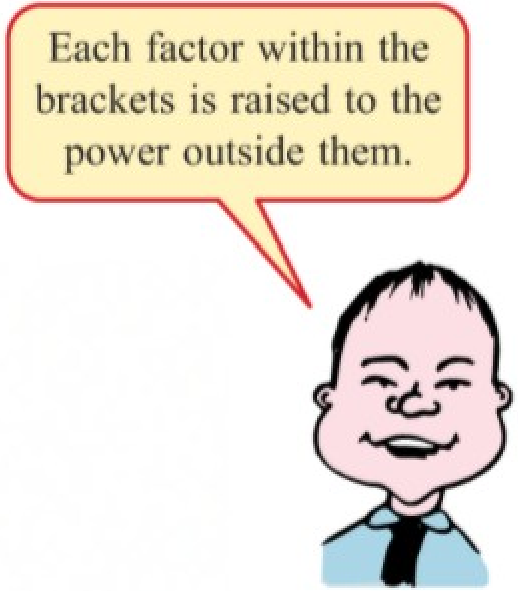
a

$$\begin{aligned}&(2x)^3\\&= 2^3 \times x^3\\&= 8x^3\end{aligned}$$

b

$$\begin{aligned}&\left(\frac{3c}{b}\right)^4\\&= \frac{3^4 \times c^4}{b^4}\\&= \frac{81c^4}{b^4}\end{aligned}$$


Each factor within the brackets is raised to the power outside them.

3 Remove the brackets of:

a $(xy)^2$

b $(ab)^3$

c $(xyz)^2$

d $(3b)^3$

e $(5a)^4$

f $(10xy)^5$

g $\left(\frac{p}{q}\right)^2$

h $\left(\frac{m}{n}\right)^3$

4 Simplify the following expressions using one or more of the index laws:

a $4b^2 \times 2b^3$

b $\dfrac{a^6b^3}{a^4b}$

c $3ab^2 \times 2a^3$

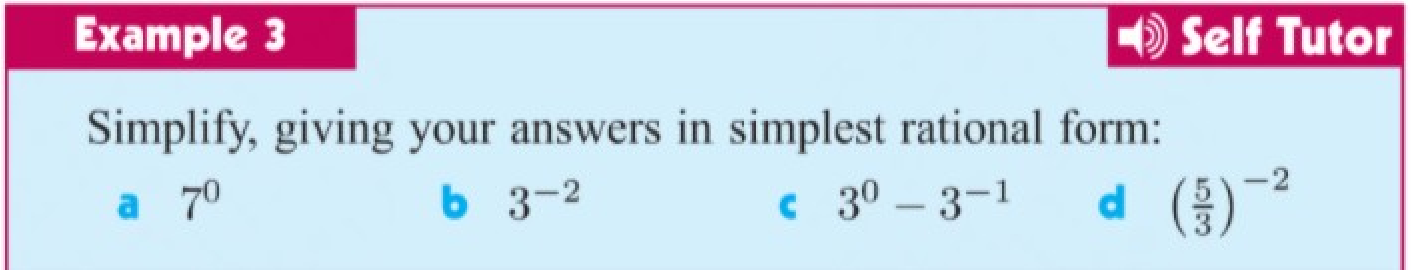
d $\dfrac{5x^3y^2}{15xy}$

e $\left(\dfrac{a^2}{5b}\right)^3$

f $\dfrac{24t^6r^4}{15t^6r^2}$

g $\dfrac{(4c^3d^2)^2}{c^2d}$

h $\dfrac{10k^7}{(2k)^5}$



Example 3

Self Tutor

Simplify, giving your answers in simplest rational form:

a 7^0 **b** 3^{-2} **c** $3^0 - 3^{-1}$ **d** $\left(\frac{5}{3}\right)^{-2}$

Example 3 **Self Tutor**

Simplify, giving your answers in simplest rational form:

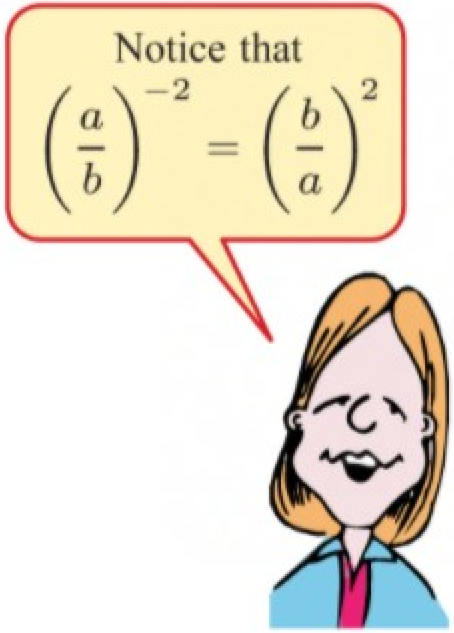
a 7^0 **b** 3^{-2} **c** $3^0 - 3^{-1}$ **d** $\left(\frac{5}{3}\right)^{-2}$

a $7^0 = 1$

b $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

c $3^0 - 3^{-1} = 1 - \frac{1}{3} = \frac{2}{3}$

d $\left(\frac{5}{3}\right)^{-2} = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$


Notice that $\left(\frac{a}{b}\right)^{-2} = \left(\frac{b}{a}\right)^2$

5 Simplify, giving your answers in simplest rational form:

a 3^0 **b** 6^{-1} **c** 4^{-1} **d** 5^0

e 4^2 **f** 4^{-2} **g** 5^3 **h** 5^{-3}

i 7^2 **j** 7^{-2} **k** 10^3 **l** 10^{-3}

6 Simplify, giving your answers in simplest rational form:

a $\left(\frac{1}{2}\right)^0$ **b** $\frac{5^4}{5^4}$ **c** $2t^0$ **d** $(2t)^0$

e 7^0 **f** 3×4^0 **g** $\frac{5^3}{5^5}$ **h** $\frac{2^6}{2^{10}}$

i $\frac{x^4}{x^9}$ **j** $\left(\frac{3}{8}\right)^{-1}$ **k** $\left(\frac{2}{3}\right)^{-1}$ **l** $\left(\frac{1}{5}\right)^{-1}$

m $2^0 + 2^1$ **n** $5^0 - 5^{-1}$ **o** $3^0 + 3^1 - 3^{-1}$ **p** $\left(\frac{1}{3}\right)^{-2}$

q $\left(\frac{2}{3}\right)^{-3}$ **r** $\left(1\frac{1}{2}\right)^{-3}$ **s** $\left(\frac{4}{5}\right)^{-2}$ **t** $\left(2\frac{1}{2}\right)^{-2}$

7 Write the following without brackets or negative indices:

a $(3b)^{-1}$

b $3b^{-1}$

c $7a^{-1}$

d $(7a)^{-1}$

e $\left(\frac{1}{t}\right)^{-2}$

f $\left(\frac{3x}{y}\right)^{-1}$

g $(5t)^{-2}$

h $(5t^{-2})^{-1}$

i xy^{-1}

j $(xy)^{-1}$

k xy^{-3}

l $(xy)^{-3}$

m $(3pq)^{-1}$

n $3(pq)^{-1}$

o $3pq^{-1}$

p $\frac{(xy)^3}{y^{-2}}$

q $(5x^{-2}y^3)^3$

r $\left(\frac{c}{2d^3}\right)^{-2}$

s $\left(\frac{3r^{-3}}{t}\right)^{-2}$

t $\left(\frac{2p}{5q^{-2}}\right)^{-3}$

8 Use the index laws to show that, for positive a and b, and integer n:

a $\dfrac{1}{a^{-n}} = a^n$

b $\left(\dfrac{a}{b}\right)^{-n} = \dfrac{b^n}{a^n}$

9 The units for speed *kilometres per hour* can be written as km/h or km h^{-1}.
Write these units in index form:

a m/s

b cubic metres/hour

c square centimetres per second

d cubic centimetres per minute

e grams per second

f kilogram metres per second

g metres per second per second.

10 Find the smaller of 2^{125} and 3^{75} without a calculator.
Hint: $2^{125} = (2^5)^{25}$

11 Order the following numbers from smallest to largest: $2^{90},\ 3^{60},\ 5^{36},\ 10^{24}$.

EXERCISE 1A

1 **a** 3^7 **b** x^9 **c** x^{5+n} **d** t^{12}

e 7^4 **f** x^4 **g** t^{6-x} **h** t^{3m-1}

i 5^6 **j** t^{12} **k** y^{3m} **l** a^{12m}

2 **a** 11^2 **b** 2^5 **c** 3^4 **d** 2^4

e 5^4 **f** 7^{t+2} **g** 3^{a-2} **h** 2^{3p-2}

i 7^2 **j** 3^{2-x} **k** 5^{4t} **l** 2^{3k-12}

m 2^{2a-b} **n** 2^{3x-4y} **o** 5^{2x+4} **p** 3^6

3 **a** x^2y^2 **b** a^3b^3 **c** $x^2y^2z^2$ **d** $27b^3$ **e** $625a^4$

f $100\,000x^5y^5$ **g** $\dfrac{p^2}{q^2}$ **h** $\dfrac{m^3}{n^3}$ **i** $\dfrac{x^4}{81}$

j $\dfrac{125}{z^3}$ **k** $\dfrac{16a^4}{b^4}$ **l** $\dfrac{27x^3}{64y^3}$

4 **a** $8b^5$ **b** a^2b^2 **c** $6a^4b^2$ **d** $\frac{x^2y}{3}$

e $\frac{a^6}{125b^3}$ **f** $\frac{8r^2}{5}$ **g** $16c^4d^3$ **h** $\frac{5k^2}{16}$

5 **a** 1 **b** $\frac{1}{6}$ **c** $\frac{1}{4}$ **d** 1 **e** 16 **f** $\frac{1}{16}$

g 125 **h** $\frac{1}{125}$ **i** 49 **j** $\frac{1}{49}$ **k** 1000 **l** $\frac{1}{1000}$

6 **a** 1 **b** 1 **c** $2, \ t \neq 0$ **d** $1, \ t \neq 0$

e 1 **f** 3 **g** $\frac{1}{25}$ **h** $\frac{1}{16}$ **i** $\frac{1}{x^5}$ **j** $\frac{8}{3}$

k $\frac{3}{2}$ **l** 5 **m** 3 **n** $\frac{4}{5}$ **o** $\frac{11}{3}$ **p** 9

q $\frac{27}{8}$ **r** $\frac{8}{27}$ **s** $\frac{25}{16}$ **t** $\frac{4}{25}$

7 **a** $\frac{1}{3b}$ **b** $\frac{3}{b}$ **c** $\frac{7}{a}$ **d** $\frac{1}{7a}$ **e** t^2

f $\frac{y}{3x}$ **g** $\frac{1}{25t^2}$ **h** $\frac{t^2}{5}$ **i** $\frac{x}{y}$ **j** $\frac{1}{xy}$

k $\frac{x}{y^3}$ **l** $\frac{1}{x^3y^3}$ **m** $\frac{1}{3pq}$ **n** $\frac{3}{pq}$ **o** $\frac{3p}{q}$

p x^3y^5 **q** $\frac{125y^9}{x^6}$ **r** $\frac{4d^6}{c^2}$ **s** $\frac{r^6t^2}{9}$ **t** $\frac{125}{8p^3q^6}$

8 **a**
$$\begin{aligned}\frac{1}{a^{-n}} &= \frac{a^0}{a^{-n}} \\ &= a^{0-(-n)} \\ &= a^{0+n} \\ &= a^n\end{aligned}$$

b
$$\begin{aligned}\left(\frac{a}{b}\right)^{-n} &= \left[\left(\frac{a}{b}\right)^{-1}\right]^n \\ &= \left(\frac{b}{a}\right)^n \\ &= \frac{b^n}{a^n}\end{aligned}$$

9 **a** $\mathrm{m\,s^{-1}}$ **b** $\mathrm{m^3\,h^{-1}}$ **c** $\mathrm{cm^2\,s^{-1}}$ **d** $\mathrm{cm^3\,min^{-1}}$
e $\mathrm{g\,s^{-1}}$ **f** $\mathrm{kg\,m\,s^{-1}}$ **g** $\mathrm{m\,s^{-2}}$

10 $2^{125} = (2^5)^{25} = 32^{25}$ and $3^{75} = (3^3)^{25} = 27^{25}$

As $27^{25} < 32^{25}$, 3^{75} is smaller.

11 The indices have a common factor of 6.

$$2^{90} = (2^{15})^6 = 32\,768^6$$
$$3^{60} = (3^{10})^6 = 59\,049^6$$
$$5^{36} = (5^6)^6 = 15\,625^6$$
$$10^{24} = (10^4)^6 = 10\,000^6$$

Order: 10^{24}, 5^{36}, 2^{90}, 3^{60}